

Dimensionado en telefonía

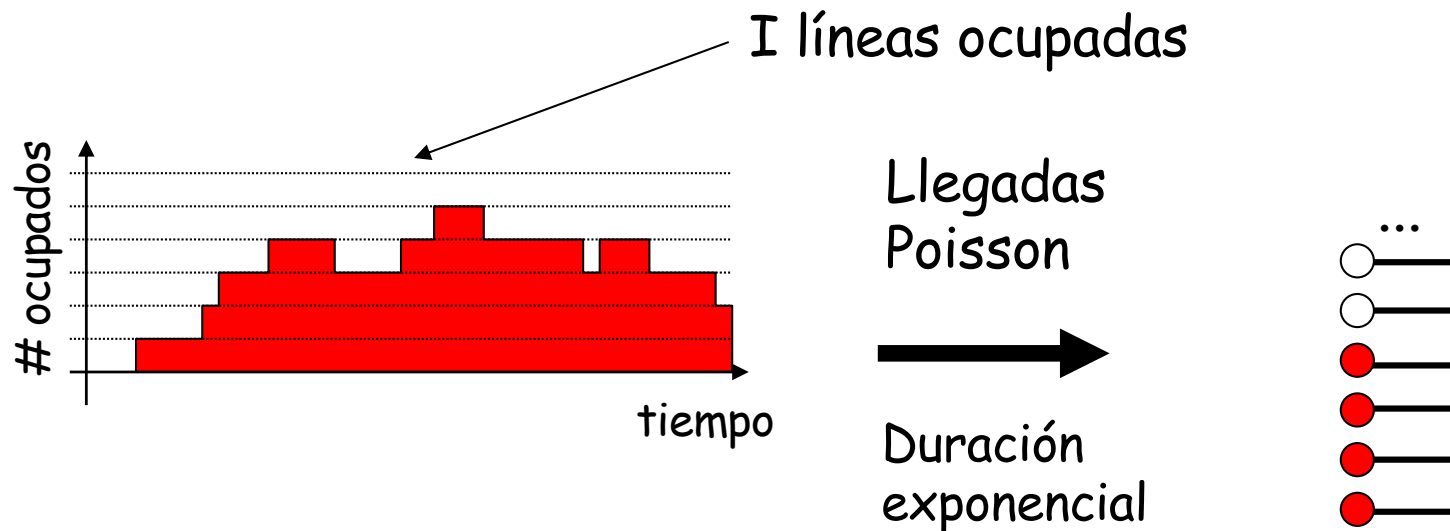
Area de Ingeniería Telemática
<http://www.tlm.unavarra.es>

Arquitectura de Redes, Sistemas y Servicios

Erlang-B

Comportamiento

- Llegadas según proceso de Poisson de tasa λ
- Duración exponencial de media s
- Número de servidores (líneas ocupados) en cada instante de tiempo es aleatorio (I)



Problemas de interés

- ¿Cuál es la probabilidad de que una llamada encuentre el sistema ocupado ?
- ¿Cuál es el número de líneas necesarias para una probabilidad objetivo ?
- ¿Cuál es el tráfico que atraviesa ese sistema y forma la carga del siguiente sistema ?

Probabilidad de bloqueo

- En un sistema con
 - Llegadas Poisson(λ)
 - Duraciones Exp($1/s$)
 - Tráfico de entrada $a = \lambda s$ (Erlangs)
 - k servidores
 - Las llamadas que llegan al sistema bloqueado se pierden
 - Probabilidad de bloqueo: ¿Cuál es $P[I=k]$? (...)
- $P[I=k] = B(a, k)$
- $B(a, k)$ es conocida como función B de Erlang (o ErlangB)
- Válida con cualquier distribución de tiempo de servicio (i.i.d.)
- Tablas:

<http://www.itu.int/itudoc/itu-d/dept/psp/ssb/planitu/plandoc/erlangt.pdf>

B de Erlang

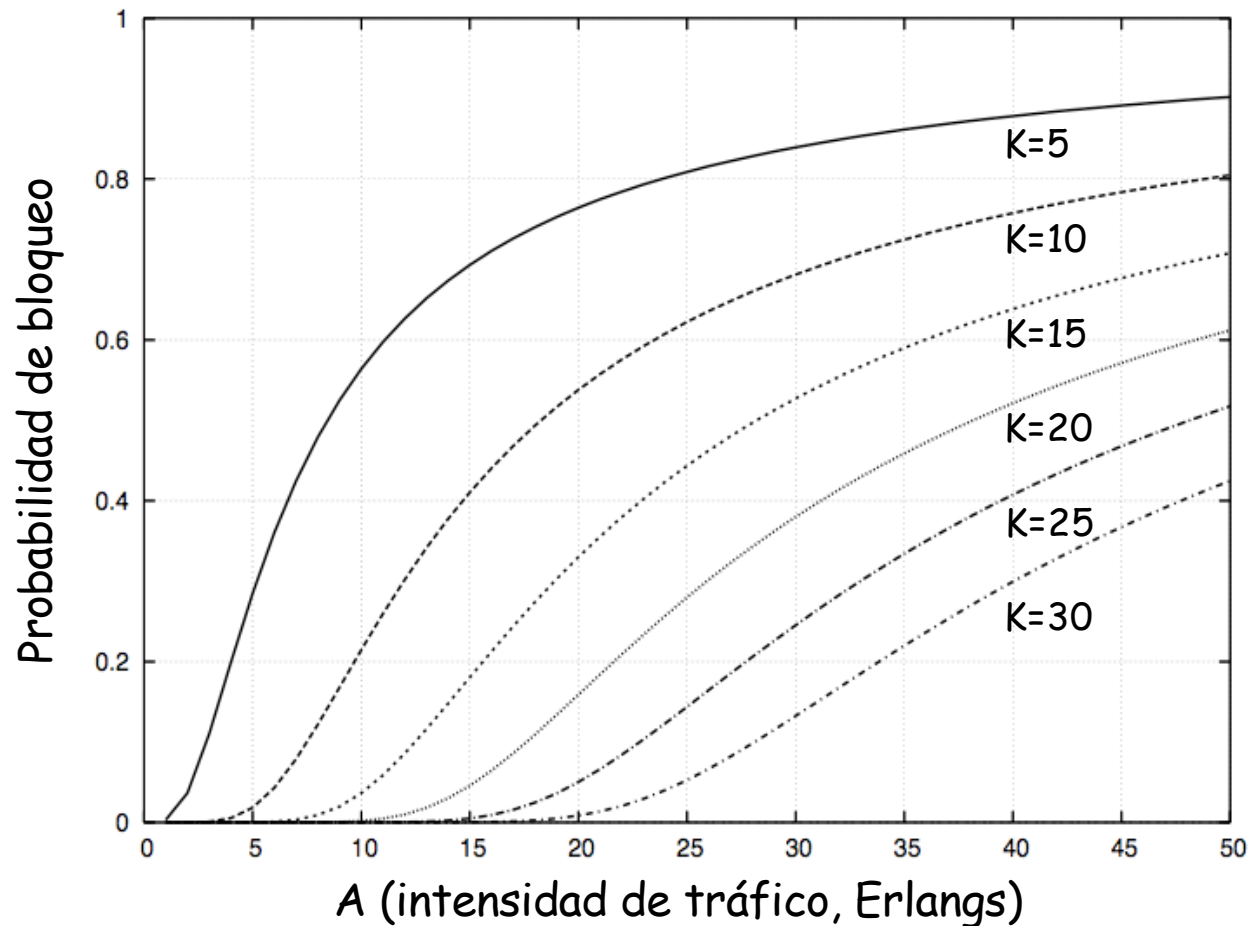
- Fórmula:

$$B(A, k) = \frac{\frac{A^k}{k!}}{\sum_{i=0}^k \frac{A^i}{i!}}$$

- Cálculo recursivo:

$$B(A, 0) = 1$$

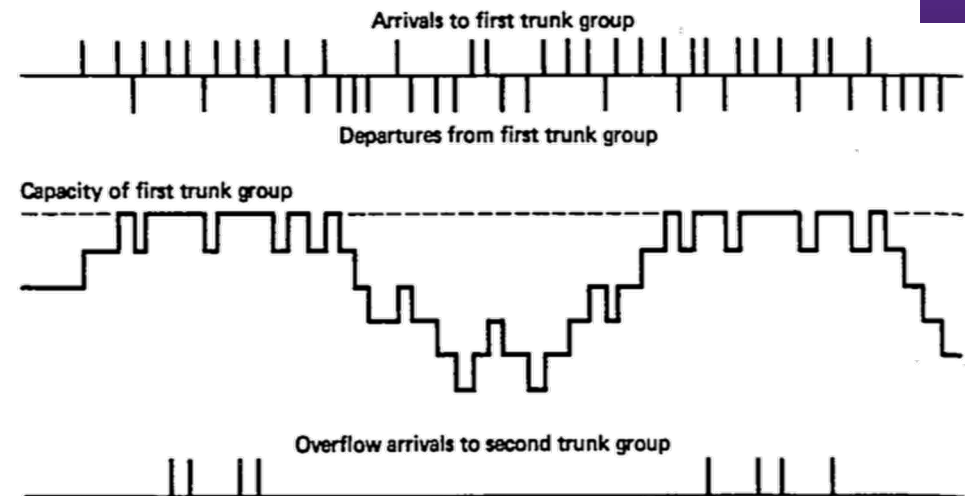
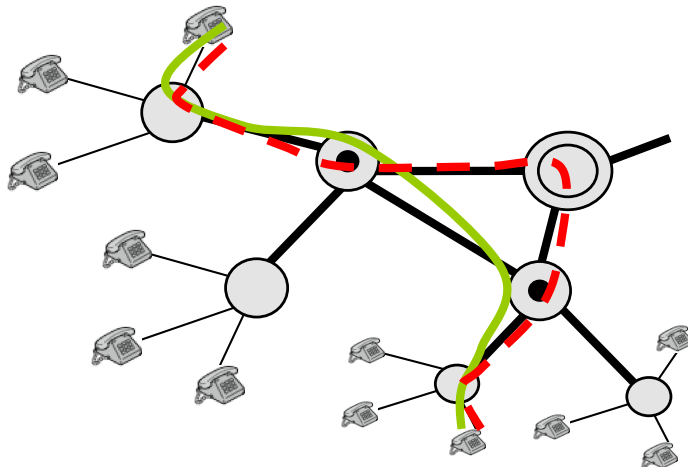
$$B(A, j) = \frac{A \cdot B(A, j-1)}{A \cdot B(A, j-1) + j}$$



Tráfico de desbordamiento

Tráfico de desbordamiento

- Tráfico que no puede ser cursado por el camino principal (por bloqueo)
- Se “desborda” (overflow) a una ruta secundaria
- Un proceso de Poisson del que se eliminan aleatoriamente (iid) muestras con probabilidad p sigue siendo un proceso de Poisson, pero con menor tasa ($p\lambda$)
- En nuestro caso las llamadas desbordadas suelen ir en bloques
- Eso da mayores probabilidades de bloqueo que con un proceso de Poisson de igual media
- Se aproxima con un proceso de Poisson de mayor tasa
- (En los problemas, en caso de no disponer de las tablas emplearemos Poisson de igual tasa, aunque esto es subdimensionar)



Mayor complejidad

- *¿ Qué ocurre si las llamadas se retienen hasta que sean atendidas ?*

Teoría de colas (función C de Erlang)

- *¿ Qué ocurre si tenemos en cuenta que hay un número finito (y conocido) de usuarios ?*

Fórmula de Engset

Preguntas pendientes

- *¿Y en el caso de conmutación de paquetes?*
 - Teoría de colas
 - Problemas más complicados
 - Peores aproximaciones
 - Mayor número de problemas sin resolver

Tablas Erlang-B

Erlang B

- Cálculos mediante software o tablas
- <http://www.itu.int/itudoc/itu-d/dept/psp/ssb/planitu/plandoc/erlangt.pdf>

n = 1 - 51

Offered traffic flow A in erlang

n	Loss probability (E)										n
	0.00001	0.00005	0.0001	0.0005	0.001	0.002	0.003	0.004	0.005	0.006	
1	.00001	.00005	.00010	.00050	.00100	.00200	.00301	.00402	.00503	.00604	1
2	.00448	.01005	.01425	.03213	.04576	.06534	.08064	.09373	.10540	.11608	2
3	.03980	.06849	.08683	.15170	.19384	.24872	.28851	.32099	.34900	.37395	3
4	.12855	.19554	.23471	.36236	.43927	.53503	.60209	.65568	.70120	.74124	4
5	.27584	.38851	.45195	.64857	.76212	.89986	.99446	1.0692	1.1320	1.1870	5
6	.47596	.63923	.72826	.99567	1.1459	1.3252	1.4468	1.5421	1.6218	1.6912	6
7	.72378	.93919	1.0541	1.3922	1.5786	1.7984	1.9463	2.0614	2.1575	2.2408	7
8	1.0133	1.2816	1.4219	1.8298	2.0513	2.3106	2.4837	2.6181	2.7299	2.8266	8
9	1.3391	1.6595	1.8256	2.3016	2.5575	2.8549	3.0526	3.2057	3.3326	3.4422	9
10	1.6970	2.0689	2.2601	2.8028	3.0920	3.4265	3.6480	3.8190	3.9607	4.0829	10
11	2.0849	2.5059	2.7216	3.3294	3.6511	4.0215	4.2661	4.4545	4.6104	4.7447	11
12	2.4958	2.9671	3.2072	3.8781	4.2314	4.6368	4.9038	5.1092	5.2789	5.4250	12
13	2.9294	3.4500	3.7136	4.4465	4.8306	5.2700	5.5588	5.7807	5.9638	6.1214	13
14	3.3834	3.9523	4.2388	5.0324	5.4464	5.9190	6.2291	6.4670	6.6632	6.8320	14
15	3.8559	4.4721	4.7812	5.6339	6.0772	6.5822	6.9130	7.1665	7.3755	7.5552	15
16	4.3453	5.0079	5.3390	6.2496	6.7215	7.2582	7.6091	7.8780	8.0995	8.2898	16
17	4.8502	5.5583	5.9110	6.8782	7.3781	7.9457	8.3164	8.6003	8.8340	9.0347	17
18	5.3693	6.1220	6.4959	7.5186	8.0459	8.6437	9.0339	9.3324	9.5780	9.7889	18
19	5.9016	6.6980	7.0927	8.1698	8.7239	9.3515	9.7606	10.073	10.331	10.552	19
20	6.4460	7.2854	7.7005	8.8310	9.4115	10.068	10.496	10.823	11.092	11.322	20
21	7.0017	7.8834	8.3186	9.5014	10.108	10.793	11.239	11.580	11.860	12.100	21
22	7.5680	8.4926	8.9462	10.180	10.812	11.525	11.989	12.344	12.635	12.885	22
23	8.1443	9.1095	9.5826	10.868	11.524	12.265	12.746	13.114	13.416	13.676	23
24	8.7298	9.7351	10.227	11.562	12.243	13.011	13.510	13.891	14.204	14.472	24
25	9.3240	10.369	10.880	12.264	12.969	13.763	14.279	14.673	14.997	15.274	25
26	9.9265	11.010	11.540	12.972	13.701	14.522	15.054	15.461	15.795	16.081	26
27	10.537	11.659	12.207	13.686	14.439	15.285	15.835	16.254	16.598	16.893	27
28	11.154	12.314	12.880	14.406	15.182	16.054	16.620	17.051	17.406	17.709	28
29	11.779	12.976	13.560	15.132	15.930	16.828	17.410	17.853	18.218	18.530	29
30	12.417	13.644	14.246	15.863	16.684	17.606	18.204	18.660	19.034	19.355	30
31	13.054	14.318	14.937	16.599	17.442	18.389	19.002	19.470	19.854	20.183	31
32	13.697	14.998	15.633	17.340	18.205	19.176	19.805	20.284	20.678	21.015	32
33	14.346	15.682	16.335	18.085	18.972	19.966	20.611	21.102	21.505	21.850	33
34	15.001	16.372	17.041	18.835	19.743	20.761	21.421	21.923	22.336	22.689	34
35	15.660	17.067	17.752	19.589	20.517	21.559	22.234	22.748	23.169	23.531	35
36	16.325	17.766	18.468	20.347	21.296	22.361	23.050	23.575	24.006	24.376	36
37	16.995	18.470	19.188	21.108	22.078	23.166	23.870	24.406	24.846	25.223	37
38	17.669	19.178	19.911	21.873	22.864	23.974	24.692	25.240	25.689	26.074	38
39	18.348	19.890	20.640	22.642	23.652	24.785	25.518	26.076	26.534	26.926	39
40	19.031	20.606	21.372	23.414	24.444	25.599	26.346	26.915	27.382	27.782	40
41	19.718	21.326	22.107	24.189	25.239	26.416	27.177	27.756	28.232	28.640	41

Erlang B

Part I, Table of A

Table of A for given values of $E_{l,n}(A) = E$ and n . See pages 3 - 14

In part I the offered traffic flow a is tabulated for given values of the loss probability $E_{l,n} = E$, and the number of devices n .

The loss probability E has the following constant values : 0.00001, 0.00005, 0.0001, 0.0005, 0.001, 0.002, 0.003, 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, and 0.4.

$n = 1 - 301$

$n = 51 - 101$

Offered traffic flow A in erlang										
Loss probability (E)										
n	0.00001	0.00005	0.0001	0.0005	0.001	0.002	0.003	0.004	0.005	0.006
51	26.789	28.706	29.632	32.094	33.332	34.716	35.611	36.293	36.852	37.331
52	27.513	29.459	30.400	32.898	34.153	35.558	36.466	37.157	37.724	38.211
53	28.241	30.216	31.170	33.704	34.977	36.401	37.322	38.023	38.598	39.091
54	28.971	30.975	31.942	34.512	35.803	37.247	38.180	38.891	39.474	39.973
55	29.703	31.736	32.717	35.322	36.631	38.094	39.040	39.760	40.351	40.857
56	30.438	32.500	33.494	36.134	37.460	38.942	39.901	40.630	41.229	41.742
57	31.176	33.266	34.273	36.948	38.291	39.793	40.763	41.502	42.109	42.629
58	31.916	34.034	35.055	37.764	39.124	40.645	41.628	42.376	42.990	43.516
59	32.659	34.804	35.838	38.581	39.959	41.498	42.493	43.251	43.873	44.406
60	33.404	35.577	36.623	39.401	40.795	42.353	43.360	44.127	44.757	45.296
61	34.151	36.351	37.411	40.222	41.633	43.210	44.229	45.005	45.642	46.188
62	34.900	37.127	38.200	41.045	42.472	44.068	45.099	45.884	46.528	47.081
63	35.651	37.906	38.991	41.869	43.313	44.927	45.970	46.764	47.416	47.975
64	36.405	38.686	39.784	42.695	44.156	45.788	46.843	47.646	48.305	48.870
65	37.160	39.468	40.579	43.523	45.000	46.650	47.716	48.528	49.195	49.766
66	37.918	40.252	41.375	44.352	45.845	47.513	48.591	49.412	50.086	50.664
67	38.677	41.038	42.173	45.183	46.692	48.378	49.467	50.297	50.978	51.562
68	39.439	41.825	42.973	46.015	47.540	49.243	50.345	51.183	51.872	52.462
69	40.202	42.615	43.775	46.848	48.389	50.110	51.223	52.071	52.766	53.362
70	40.967	43.405	44.578	47.683	49.239	50.979	52.103	52.959	53.662	54.264
71	41.734	44.198	45.382	48.519	50.091	51.848	52.984	53.848	54.558	55.166
72	42.502	44.992	46.188	49.357	50.944	52.718	53.865	54.739	55.455	56.070
73	43.273	45.787	46.996	50.195	51.799	53.590	54.748	55.630	56.354	56.974
74	44.045	46.585	47.805	51.035	52.654	54.463	55.632	56.522	57.253	57.880
75	44.818	47.383	48.615	51.877	53.511	55.337	56.517	57.415	58.153	58.786
76	45.593	48.183	49.427	52.719	54.369	56.211	57.402	58.310	59.054	59.693
77	46.370	48.985	50.240	53.563	55.227	57.087	58.289	59.205	59.956	60.601
78	47.149	49.787	51.054	54.408	56.087	57.964	59.177	60.101	60.859	61.510
79	47.928	50.592	51.870	55.254	56.948	58.842	60.065	60.998	61.763	62.419
80	48.710	51.397	52.687	56.101	57.810	59.720	60.955	61.895	62.668	63.330
81	49.492	52.204	53.506	56.949	58.673	60.600	61.845	62.794	63.573	64.241
82	50.277	53.012	54.325	57.798	59.537	61.480	62.737	63.693	64.479	65.153
83	51.062	53.822	55.146	58.649	60.403	62.362	63.629	64.594	65.386	66.065
84	51.849	54.633	55.968	59.500	61.269	63.244	64.522	65.495	66.294	66.979
85	52.637	55.445	56.791	60.352	62.135	64.127	65.415	66.396	67.202	67.893
86	53.427	56.258	57.615	61.206	63.003	65.011	66.310	67.299	68.111	68.808
87	54.218	57.072	58.441	62.060	63.872	65.897	67.205	68.202	69.021	69.724
88	55.010	57.887	59.267	62.915	64.742	66.782	68.101	69.106	69.932	70.640
89	55.804	58.704	60.095	63.772	65.612	67.669	68.998	70.011	70.843	71.557
90	56.598	59.526	60.923	64.629	66.484	68.556	69.896	70.917	71.755	72.474
91	57.394	60.344	61.753	65.487	67.356	69.444	70.794	71.823	72.668	73.393
92	58.192	61.164	62.584	66.346	68.229	70.333	71.693	72.730	73.581	74.311
93	58.990	61.985	63.416	67.206	69.103	71.222	72.593	73.637	74.495	75.231
94	59.789	62.807	64.248	68.067	69.978	72.113	73.493	74.545	75.410	76.151
95	60.590	63.630	65.082	68.928	70.853	73.004	74.394	75.454	76.325	77.072
96	61.392	64.454	65.917	69.791	71.729	73.896	75.296	76.364	77.241	77.993
97	62.194	65.279	66.752	70.654	72.606	74.788	76.199	77.274	78.157	78.915
98	62.998	66.105	67.589	71.518	73.484	75.681	77.102	78.185	79.074	79.837
99	63.803	66.932	68.426	72.383	74.363	76.575	78.006	79.096	80.090	80.760
100	64.609	67.760	69.265	73.248	75.242	77.469	78.910	80.008	80.910	81.684
101	65.416	68.589	70.104	74.115	76.122	78.364	79.815	80.920	81.829	82.608
n	0.00001	0.00005	0.0001	0.0005	0.001	0.002	0.003	0.004	0.005	0.006

Erlang B

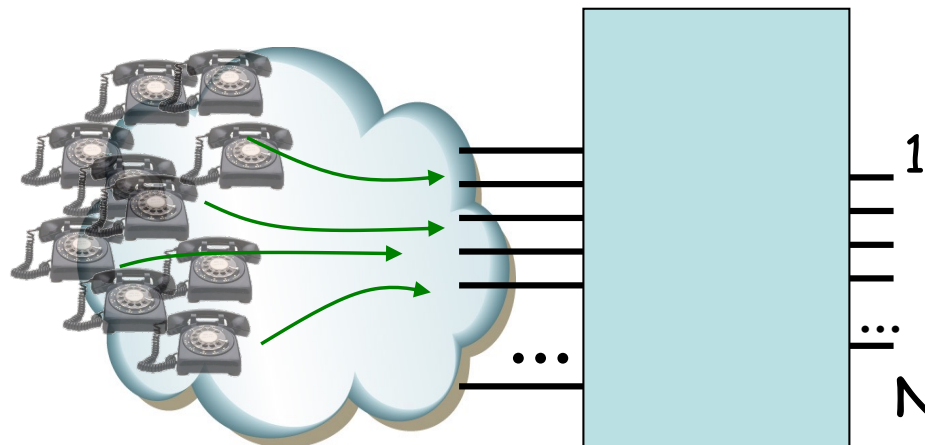
n = 51 - 101

Example

- Find the required number of devices n for $A = 60$ erlang and the loss probability $E = 0.001$.
- From page 5 and the column for $E = 0.001$ can be seen that $n = 83$ corresponds to the A - value 60.403 erlang and $n = 82$ to $A = 59.537$.
- Consequently, the required number of devices is 83.

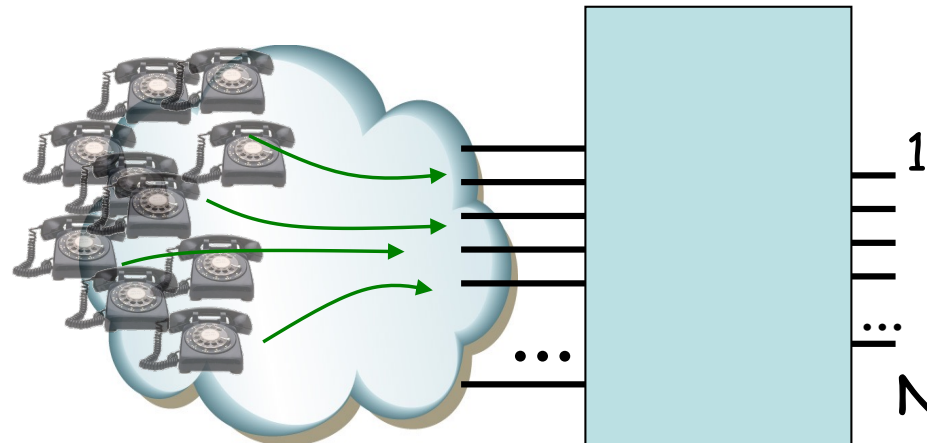
n	Offered traffic flow A in erlang										n
	Loss probability (E)										
	0.00001	0.00005	0.0001	0.0005	0.001	0.002	0.003	0.004	0.005	0.006	
51	26.789	28.706	29.632	32.094	33.332	34.716	35.611	36.293	36.852	37.331	51
52	27.513	29.459	30.400	32.898	34.153	35.558	36.466	37.157	37.724	38.211	52
53	28.241	30.216	31.170	33.704	34.977	36.401	37.322	38.023	38.598	39.091	53
54	28.971	30.975	31.942	34.512	35.803	37.247	38.180	38.891	39.474	39.973	54
55	29.703	31.736	32.717	35.322	36.631	38.094	39.040	39.760	40.351	40.857	55
56	30.438	32.500	33.494	36.134	37.460	38.942	39.901	40.630	41.229	41.742	56
57	31.176	33.266	34.273	36.948	38.291	39.793	40.763	41.502	42.109	42.629	57
58	31.916	34.034	35.055	37.764	39.124	40.645	41.628	42.376	42.990	43.516	58
59	32.659	34.804	35.838	38.581	39.959	41.498	42.493	43.251	43.873	44.406	59
60	33.404	35.577	36.623	39.401	40.795	42.353	43.360	44.127	44.757	45.296	60
61	34.151	36.351	37.411	40.222	41.633	43.210	44.229	45.005	45.642	46.188	61
62	34.900	37.127	38.200	41.045	42.472	44.068	45.099	45.884	46.528	47.081	62
63	35.651	37.906	38.991	41.869	43.313	44.927	45.970	46.764	47.416	47.975	63
64	36.405	38.686	39.784	42.695	44.156	45.788	46.843	47.646	48.305	48.870	64
65	37.160	39.468	40.579	43.523	45.000	46.650	47.716	48.528	49.195	49.766	65
66	37.918	40.252	41.375	44.352	45.845	47.513	48.591	49.412	50.086	50.664	66
67	38.677	41.038	42.173	45.183	46.692	48.378	49.467	50.297	50.978	51.562	67
68	39.439	41.825	42.973	46.015	47.540	49.243	50.345	51.183	51.872	52.462	68
69	40.202	42.615	43.775	46.848	48.389	50.110	51.223	52.071	52.766	53.362	69
70	40.967	43.405	44.578	47.683	49.239	50.979	52.103	52.959	53.662	54.264	70
71	41.734	44.198	45.382	48.519	50.091	51.848	52.984	53.848	54.558	55.166	71
72	42.502	44.992	46.188	49.357	50.944	52.718	53.865	54.739	55.455	56.070	72
73	43.273	45.787	46.996	50.195	51.799	53.590	54.748	55.630	56.354	56.974	73
74	44.045	46.585	47.805	51.035	52.654	54.463	55.632	56.522	57.253	57.880	74
75	44.818	47.383	48.615	51.877	53.511	55.337	56.517	57.415	58.153	58.786	75
76	45.593	48.183	49.427	52.719	54.369	56.211	57.402	58.310	59.054	59.693	76
77	46.370	48.985	50.240	53.563	55.227	57.087	58.289	59.205	59.956	60.601	77
78	47.149	49.787	51.054	54.408	56.087	57.964	59.177	60.101	60.859	61.510	78
79	47.928	50.592	51.870	55.254	56.948	58.842	60.065	60.998	61.763	62.419	79
80	48.710	51.397	52.687	56.101	57.810	59.720	60.955	61.895	62.668	63.330	80
81	49.492	52.204	53.506	56.949	58.673	60.600	61.845	62.794	63.573	64.241	81
82	50.277	53.012	54.325	57.798	59.537	61.480	62.737	63.693	64.479	65.153	82
83	51.062	53.822	55.146	58.649	60.403	62.362	63.629	64.594	65.386	66.065	83
84	51.849	54.633	55.968	59.500	61.269	63.244	64.522	65.495	66.294	66.979	84
85	52.637	55.445	56.791	60.352	62.135	64.127	65.415	66.396	67.202	67.893	85
86	53.427	56.258	57.615	61.206	63.003	65.011	66.310	67.299	68.111	68.808	86
87	54.218	57.072	58.441	62.060	63.872	65.897	67.205	68.202	69.021	69.724	87
88	55.010	57.887	59.267	62.915	64.742	66.782	68.101	69.106	69.932	70.640	88
89	55.804	58.704	60.095	63.772	65.612	67.669	68.998	70.011	70.843	71.557	89
90	56.598	59.526	60.923	64.629	66.484	68.556	69.896	70.917	71.755	72.474	90
91	57.394	60.344	61.753	65.487	67.356	69.444	70.794	71.823	72.668	73.393	91
92	58.192	61.164	62.584	66.346	68.229	70.333	71.693	72.730	73.581	74.311	92
93	58.990	61.985	63.416	67.206	69.103	71.222	72.593	73.637	74.495	75.231	93
94	59.789	62.807	64.248	68.067	69.978	72.113	73.493	74.545	75.410	76.151	94
95	60.590	63.630	65.082	68.928	70.853	73.004	74.394	75.454	76.325	77.072	95
96	61.392	64.454	65.917	69.791	71.729	73.896	75.296	76.364	77.241	77.993	96
97	62.194	65.279	66.752	70.654	72.606	74.788	76.199	77.274	78.157	78.915	97

Ejercicio



Enunciado

- 250 usuarios de telefonía de la misma central local
- En la hora cargada:
 - 200 hacen (cada uno) en media 1 llamada/hora de 3 min
 - 50 hacen (cada uno) 2.5 llamadas/hora de 5 min
- La central tiene un enlace E1
- Calcule la probabilidad de bloqueo en la hora cargada



Cálculo de la Erlang-B

Cálculo recursivo

- Fórmula:

$$B(A, k) = \frac{\frac{A^k}{k!}}{\sum_{i=0}^k \frac{A^i}{i!}}$$

- Cálculo recursivo:

$$B(A, 0) = 1$$

$$B(A, j) = \frac{A \cdot B(A, j-1)}{A \cdot B(A, j-1) + j}$$

$$B(A, j) = \frac{A \cdot B(A, j-1)}{A \cdot B(A, j-1) + j} = \frac{A \cdot \frac{\frac{A^{j-1}}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!}}}{A \cdot \frac{\frac{A^{j-1}}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!}} + j} = \frac{\frac{\frac{A^j}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!}}}{\frac{\frac{A^j}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!}} + j}$$

$$B(A, j) = \frac{\frac{A^j}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!} \left(\frac{\frac{A^j}{(j-1)!}}{\sum_{i=0}^{j-1} \frac{A^i}{i!}} + j \right)} = \frac{\frac{A^j}{(j-1)!}}{\frac{A^j}{(j-1)!} + j \sum_{i=0}^{j-1} \frac{A^i}{i!}}$$

Cálculo recursivo

- Fórmula:

$$B(A, k) = \frac{A^k}{k!} / \sum_{i=0}^k A^i / i!$$

- Cálculo recursivo:

$$B(A, 0) = 1$$

$$B(A, j) = \frac{A \cdot B(A, j-1)}{A \cdot B(A, j-1) + j}$$

$$B(A, j) = \frac{\frac{A^j}{j(j-1)!}}{\frac{A^j}{j(j-1)!} + \sum_{i=0}^{j-1} A^i / i!} = \frac{\frac{A^j}{j!}}{\frac{A^j}{j!} + \sum_{i=0}^{j-1} A^i / i!} = \frac{\frac{A^j}{j!}}{\sum_{i=0}^j A^i / i!}$$

$$B(A, j) = \frac{\frac{A^j}{(j-1)!}}{\sum_{i=0}^{j-1} A^i / i! \left(\frac{A^j}{(j-1)!} / \sum_{i=0}^{j-1} A^i / i! + j \right)} = \frac{\frac{A^j}{(j-1)!}}{\frac{A^j}{(j-1)!} + j \sum_{i=0}^{j-1} A^i / i!}$$

Programa de cálculo

- Fórmula:
$$B(A,k) = \frac{A^k}{\sum_{i=0}^k \frac{A^i}{i!}}$$

- Cálculo recursivo:

$$B(A,0) = 1$$

$$B(A,j) = \frac{A \cdot B(A,j-1)}{A \cdot B(A,j-1) + j}$$

- Ejemplo en Python:

```
def erlang_b(A, n):  
  
    if n == 0:  
        return 1.0  
  
    erlang_b_val = 1.0  
  
    for j in range(1, n + 1):  
        erlang_b_val = (A * erlang_b_val) / ((A * erlang_b_val) + j)  
  
    return erlang_b_val
```

Ejemplo

```
$ python
Python 3.11.6 (main, Nov 21 2023, 13:21:41) [Clang 15.0.0 (clang-1500.0.40.1)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> def erlang_b(A, n):
...
...     if n == 0:
...         return 1.0
...
...     erlang_b_val = 1.0
...
...     for j in range(1, n + 1):
...         erlang_b_val = (A * erlang_b_val) / ( (A * erlang_b_val) + j)
...
...     return erlang_b_val
...
>>>
>>> # Ejemplo de uso:
>>> trafico = 10
>>> canales = 15
>>>
>>> prob_bloqueo = erlang_b(trafico, canales)
>>> print("La probabilidad de bloqueo es: {}".format(prob_bloqueo))
La probabilidad de bloqueo es: 0.0364969454723708
```

Programa en Java

```
A = 10;  
int n = 15;  
double erlang_b_val = 1.0;
```

```
for (int j = 1; j <= n; j++) {  
    erlang_b_val = (A * erlang_b_val) / (A * erlang_b_val + j);  
}
```



En Python:

```
A = 10  
n = 15  
erlang_b_val = 1.0
```

```
for j in range(1, n + 1):  
    erlang_b_val = (A * erlang_b_val) / ( (A * erlang_b_val) + j)
```

