On the Use of Balking for Estimation of the Blocking Probability for OBS Routers with FDL Lines *

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Abstract. This paper deals with estimation of blocking probabilities for OBS switches with Fiber Delay Lines (FDLs) and full wavelength conversion. An incoming burst that finds the wavelengths occupied is temporarily stored in a FDL. Hence, contention will be sorted out successfully if the residual life of the system is smaller than the maximum FDL delay. In order to derive the blocking probability, the most accurate methodology to date is the use of *balking* systems [1–4]. Even though the approach is accurate for very short lengths of the FDLs we identify the cases in which inaccuracy is detected. This happens precisely when the system works with low loss probabilities. Mainly for large number of wavelengths on the fibers and values of the FDL length at least in the vicinity of the burst service time.

1 Introduction and Problem Statement

Optical Burst Switching (OBS) has received considerable research attention as a promising solution for all-optical transmission of data bursts. Burst Control Packets (BCPs) are transmitted through a control channel that is separated from the data channel. Hence, the data payload is transmitted entirely in the optical domain, while the control packet is processed electronically and suffers O/E/O conversion. Such BCPs are sent before the burst is actually transmitted and serve to announce the incoming bursts so that switches can be configured beforehand. However, since resources are unconfirmed, blocking may occur at any switch along the path from source to destination. Thus, OBS is a transfer mode that is halfway between circuit switching and packet switching.

Needless to say, the burst blocking probability is the primary performance measure for OBS networks. This paper is concerned with the analysis of the blocking probability for OBS switches equipped with Fiber Delay Line (FDLs). Since optical buffering is not available at the moment, nor it is a foreseeable technology that will appear in the close future, optical switch designers resort to alternate solutions such as the FDLs. An FDL is a fiber with an specified length,

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that provides a delay which is equal to the propagation time of a burst in the FDL. Due to the limited delay availability, a buffered burst may be dropped if the output port/wavelength occupation persists when the burst is to exit the FDL.

An architecture named the *Tune and Select* switch is shown in [5]. The switch features N input and output ports. We assume c wavelengths per port, with full wavelength conversion capability, i. e. bursts can be switched from any input wavelength and port to any output wavelength and port.

Assumming traffic destinations are uniform, the analysis can be focused on a single output fiber only. Regardless of the possible internal switch architecture, an abstract model for performance evaluation can be derived with c parallel servers representing the wavelengths per port and a number of fiber delay lines. In the system each FDL has a length L and a maximum storage delay of D_{max} time units. For comparison with previous works we assume the architecture uses variable-delay FDLs, JET signaling and LAUC as the scheduling algorithm [1, 6].

In this paper, we wish to study the accuracy of modeling this particular scenario of FDL-equipped OBS switches using a form of queue with impatience, named a *balking* model. The use of balking for the analysis of such FDL-equipped OBS routers has been proposed in other studies [1–4], with successful results. However, such papers considered a number of wavelengths per port relatively small and they did not check the range of the design parameters where the approximation was accurate. Alternatively, we focus on switch architectures with larger number of wavelengths. For those cases, even with a small number of FDLs the number of bursts that could be simultaneously buffered is large enough to make the length of the FDLs and not the number of fibers as the limiting factor. In this paper we show that a strong mismatch between analytical and simulation results is observed and we reveal that some assumptions about the applicability of the balking model are wrong.

Let us assume that the c wavelengths of an output port are occupied (namely the output port is blocked). An arrival to the system will not enter an FDL *if the delay provided by the FDLs is not large enough to hold the burst during the system blocking time*. In other words, an arrival will not enter the FDL if the output port residual life is larger than the delay provided by the fibers. A queueing system in which arrivals decide on whether to enter the system based on the system state (number of users, current delay, etc) is called a *balking* system or a system with *discouraged arrivals* [7, pp. 123]. For instance, an $M/M/c/K$ system falls within this category, since arrivals will not enter the system if K customers are already inside it.

The balking system lends itself as a good model for analyzing the OBS routers with FDLs. In [8] the $M/M/c/K$ is used as a lower bound. The latter model assumes that up to $K-c$ bursts may fit in the FDLs. However, the number of bursts that fit within an FDL cannot be determined beforehand. This is illustrated in Fig. 1 [2]. The number of bursts that can be accommodated in a single FDL depends on the FDL length and the burst length, which in turn depends

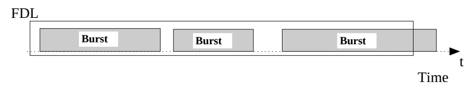


Fig. 1. The number of wavelengths in an FDL depends on the burst length distribution

on the burst length distribution. For example, [2] reports that the number of bursts per FDL can be arbitrarily large if the burst length is exponential. An improved $M/M/c/K$ approximation is presented in [9], in which K is derived as a function of the FDL length. However, it fails to provide a lower bound as the FDL length becomes larger.

The fundamental limitation of the abovementioned models is that an FDL does not behave as a queue with limited number of customers. Actually, an FDL will reject bursts if the residual life of the servers is larger than the FDL holding time, irrespective of the number of bursts already present in the FDLs. Precisely, the balking models that have been proposed use the FDL holding time as the reject criteria [1–4].

This paper contributes to the modeling of OBS routers with FDLs by pushing the limits of the balking models. The aim is to identify the limiting cases for which the balking approach is not accurate. While the models proposed in [1–4] are most valuable and serve to analyse the most common cases of FDL-equipped OBS routers, further insight into the accuracy of balking systems is provided in this paper. In fact, we show that the model is less accurate as the number of wavelengths per fiber increases. This is a serious drawback of balking models, since the foreseeable technological evolution is towards hundreds of wavelengths.

2 Analysis

First, the balking model is presented, as a continuous-time discrete Markov chain. Then, the accuracy of the model is discussed and the causes for deviation with the empirical results are identified.

2.1 A Balking Model for FDL-Equipped OBS Routers

The balking model incorporates the probability that a burst is dropped, i.e. the probability that a burst does not enter the system *because the FDL is too short to hold the burst for the system residual life*. Let $\{X_t, t > 0\}$ be a continuous-time discrete Markov chain that represents the number of bursts in the output port (c servers and FDLs). Let λ be the Poisson arrival rate and μ the service rate, $\rho = \lambda/c\mu$. Let us denote the probability that an incoming burst does not enter the system by β_k for $k = n-c, n \geq c$ where n is the system state (there is balking only for states larger than the number of wavelengths). The arrival rate in state $n > c$ is thus $\lambda_n = \lambda(1-\beta_{k-1})$, where λ is the output port arrival rate (Poisson). Being consistent with the analysis in [1–4], the service time distribution will be

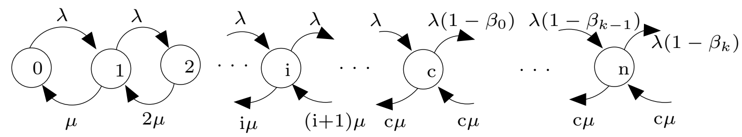


Fig. 2. $\{X_t, t > 0\}$, number of bursts in the output port

exponential. Fig. 2 shows a state diagram of $\{X_t, t > 0\}$ with the corresponding arrival and departure rates.

In order to obtain the steady state probabilities π_n, calculation of the probabilities β_k is needed. Let us assume that all wavelengths (servers) are fully occupied and consider the time that a new arrival must wait before it can be served, named the *residual life of the system (wavelengths + FDLs) in state* n, which will be denoted by T_n. Then

$$T_n = \hat{R} + \sum_{i=1}^{n-c} \hat{U}, \quad n \geq c \tag{1}$$

where $\hat{R}$ is the residual life of the wavelengths (the servers) and $\hat{U}$ is the random variable that represents the interdeparture time. Note that an arriving burst will wait for the residual life of the wavelengths plus $n - c$ departures corresponding to the bursts already in the FDLs, due to the PASTA property (Poisson Arrivals See Time Averages).

The residual life of the wavelengths is derived using the residual life of a single wavelength R_1, which is exponential due to the memoryless property.

$$P(\hat{R} > x) = \prod_{i=1}^{c} P(R_i > x) = P(R_1 > x)^c \tag{2}$$

On the other hand, the probability density function is [10, pp. 172]

$$\hat{f}_{R_i}(x) = \frac{P(X > x)}{E[X]} \tag{3}$$

which, for exponential random variables, yields $P(\hat{R} > x) = e^{-c\mu x}$.

Due to the memoryless property the interdeparture times are also exponential and the sum of $n - c$ independent exponential random variables is an Erlang random variable. Thus,

$$\beta_{n-c} = P(T_n > L) = e^{-c\mu L} \sum_{h=0}^{n-c} \frac{(c\mu L)^h}{h!} \quad n = c, c+1\ldots \tag{4}$$

The Markov chain is solved using the equilibrium equations and the steady state probabilities are

$$\pi_0 = \left(\sum_{i=0}^{c} \frac{(c\rho)^i}{i!} + \rho^c \sum_{i=c+1}^{\infty} \prod_{j=c}^{i-1} \rho(1-\beta_{j-c}) \right)^{-1}$$

$$\pi_n = \begin{cases} \pi_0 \frac{(c\rho)^k}{k!} & , \quad n < c \\ \pi_0 \frac{c^c}{c!} \rho^k \prod_{i=0}^{k-c-1} (1-\beta_i) & , \quad n \geq c \end{cases} . \tag{5}$$

Finally, the blocking probability is the ensemble average of blocking probabilities over the wavelength occupation states: $P(blocking) = \sum_{k=0}^{\infty} \pi_{k+c}\beta_k$.

This is the model that has been proposed in [1–4]. In [4] the Erlang random variable is used to model a single wavelength model ($c = 1$), with special emphasis in the study of FDLs with discrete step allowable delays. In [2, 3] the authors use the balking model presented in this section and an improvement with respect to previous models is achieved [8, 9]. However, the number of wavelengths is $c = \{2, 3\}$. In [1] no greater number than $c = 10$ is simulated. In this paper, we focus on a scenario with larger number of wavelengths since this is consistent with the foreseeable evolution of optical networks. For example, commercial CWDM routers are available with 8 wavelengths [11] and DWDM prototypes have been reported with 32 [12] and 128 wavelengths [13]. For such number of wavelengths we find discrepancies between the analytical and simulation results.

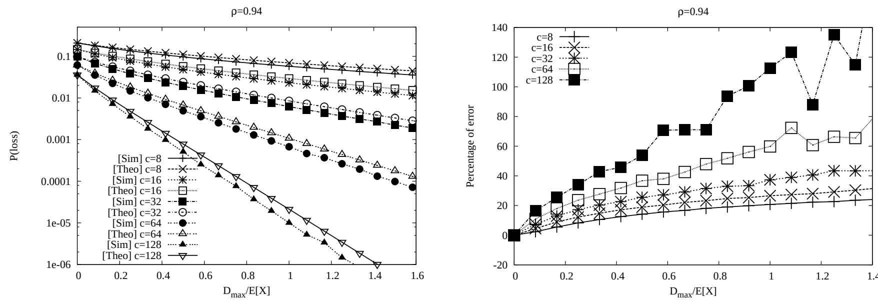
3 Results and Discussion

A simulation model has been built using the *dsim* [14] building blocks. Such simulation library has also been used in other papers [15–18]. The wavelength speed is set to 10 Gbps and the number of wavelengths c from 8 to 128.

The burst average size will be set to $15KBytes$, which is the average file size in the Internet as reported by [19], yielding a transmission time $E[X] = 12.288\mu s$. This transmission time is similar in other studies [20]. Switching times will be assumed to be negligible, since SOA-based switches achieve switching times in the vicinity of nanoseconds [21–24]. Finally, each simulation run consists of 10^8 burst arrivals.

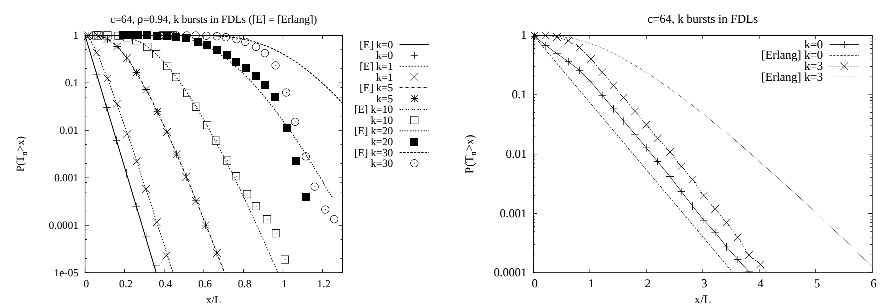
Fig. 3(a) shows the blocking probability versus the normalized FDL length (D_{max} divided by the burst transmission time), for a system with $\rho = 0.94$ and different number of wavelengths c. For $D_{max} = 0$ the blocking time can be approximated accurately by the Erlang-B formula. However, as D_{max} increases, a discrepancy with the model is detected. This is shown in Fig. 3(b) where the percentage of error in the estimation using the model with balking increases as c or D_{max} increase.

In the following subsection the detected discrepancy between the analytical and simulation models is explained in detail. It turns out that neither the probabilities β_k in equation 4 (discouraged arrivals) nor the state probabilities π_n (equation 5) accurately model our case study of OBS router with FDLs. Consequently, the balking model (Fig. 2) hypothesis are revised.



(a) (b)

Fig. 3. Burst dropping probability versus normalized FDL length for different utilization factors (Theoretical and Simulation results)



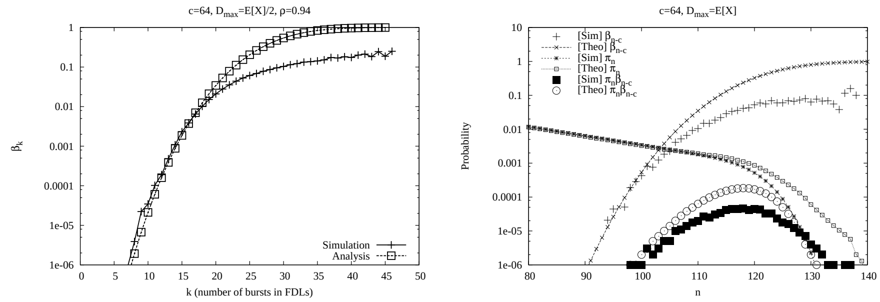
(a) $D_{max} = E[X]/2$ (b) $D_{max} = E[X]/24$

Fig. 4. Residual life survival function, experimental and theoretical

3.1 Discouraged Arrival Probability

From equation 4, the system residual life T_n can be approximated by an Erlang random variable, since interdeparture times are assumed to be exponential. Fig. 4(a) shows the residual life survival function conditioned to the number of bursts (n) in the system (FDLs+wavelengths) $P(T_n > x)$, for $D_{max} = E[X]/2$.

For $n = c$ (or $k = 0$, no bursts in FDLs), the residual life turns out to be exponential as expected. However, as the system occupancy grows larger there is a significant deviation between the Erlang approximation and the measured residual life. It is also verified by simulation that the discrepancy between the Erlang residual life and the empirical counterpart is larger for residual life values



(a) Discouraged arrival probability (β_k) with $D_{max} = E[X]/2$

(b) Comparison between the state probabilities (π_n) and the discouraged arrival probabilities (β_k)

Fig. 5. Influence of the computation of β_k on the state probabilities

close to the fiber delay. For example, Fig. 4(b) presents a comparison between analytical and empirical residual lives for $D_{max} = E[X]/24$. It shows that even for low n values, the discrepancy between the curves is significant when the residual life is close to D_{max}.

This deviation carries over to the discouraged arrival probability β_k, as shown in Fig. 5(a). As we move to states with larger number of bursts in FDLs the values of β_k obtained from the balking analytical model differ greatly from the simulation results. However, those are the low probability states of the Markov chain. We show in the next section that those states are fundamental in the final value of the loss probability.

3.2 State Probabilities (π_n)

Note that the discouraged arrival probabilities β_k play a crucial role in the calculation of the state probabilities (equation 5). Hence, the discrepancy between analitycal and empirical results in β_k carries over to the state probabilities π_n.

Fig. 5(b) shows empirical versus analytical results for both state probabilities and discouraged arrival probabilities, for a number of wavelengths equal to 64. Both values (β_k and π_n) take part in product form on the calculation of the loss probability. Fig. 5(b) also shows this product $\beta_k \pi_n$. The discrepancy in the discouraged arrival probability and state probabilities happen precisely for high occupancy states with small probabilities of occurrence. However, those are the states where losses take place. Therefore, the deviation from the analytical to the real values in that region of the state-space produces the misbehavior of the loss probability shown in Fig. 3.

3.3 Explaining the Discrepancy between Analytical and Empirical Results

The above figures show that the discrepancy between analytical and empirical results become more significant as the loss probability is decreased. Hence, the model becomes less accurate for realistic systems of WDM technology, with a higher output degree (number of wavelengths) and lower losses. This is due to the effect of the FDLs on the behavior of the system.

On a congested balking system (system state $n \geq c$), the acceptance of a new arrival depends only on the system state. Even the conditional probability β_{n-c} depends only on the state number n. However, the FDLs act as queues with a maximum delay D_{max}. On a system with FDLs, β_k *depends also on the system residual life*. Independent of the system state, the arrivals that find the system with a residual life longer than D_{max} will get lost.

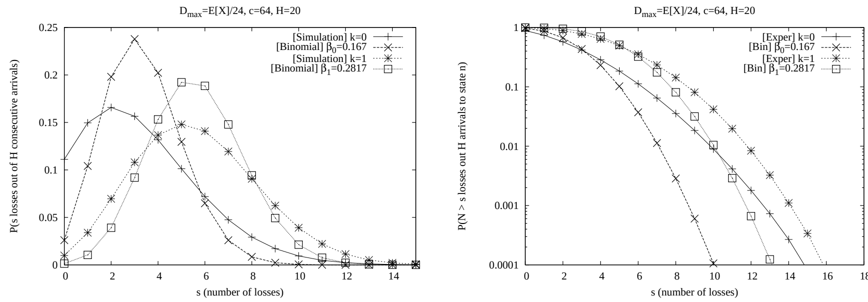
Consider the arrival process when the system is in state n where $n \geq c$. In a balking system this arrival process is sampled randomly with probability β_{n-c}. We could model the arrivals to the state as the events of a random variable with a value of 1 when the arrival is lost (probability β_{n-c}) and with value 0 when it gets into the system (probability $1 - \beta_{n-c}$). Then, the distribution of number of losses out of H consecutive arrivals would be Binomial, i. e,

$$P(s) = \binom{H}{s} \beta_{n-c}^{s} (1 - \beta_{n-c})^{H-s} \tag{6}$$

Fig. 6 shows the empirical distribution compared to the values of equation 6 for $H = 20$, taking β_{n-c} from the simulation results. Both distributions differ significantly, and this implies that sampling is not performed randomly in the original process. Actually, when the system occupancy is large, an arrival is discouraged depending on the previous arrivals. Consider a system with c bursts in service and arrivals i and $i+1$ that happen in state c and $c+1$ respectively (arrival i is accepted). The residual life of the system will depend on the size (transmission time) of arrival i, and so the event of the arrival $i+1$ being discouraged or not. Consequently, *the discouraged arrival probability does not depend on the number of bursts in the system solely, but also on the system residual life, which is a continuous random variable.* As a result, the balking Markov model (Fig. 2) cannot be applied.

4 Conclusions

In this paper we study the applicability of the balking model for FDL-equipped OBS routers. We conclude that the balking model accuracy depends on the ratio between fiber delay and service time. If the ratio is large then the balking model is not accurate to derive the blocking probability. Stronger discrepancies between analytical and simulation results are observed as the number of wavelengths per port increases. But precisely, the foreseeable technological evolution is towards hundreds of wavelengths.



(a) Mass probability function (b) Survival function

Fig. 6. Empirical probability of s discouraged arrivals out of H consecutive arrivals to system state $n = c$, compared to a binomial random variable (i.e. random sampling). $D_{max} = E[X]/24$, $c = 64$

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